

Teaching for Mastery: The What, How and the Why

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Teaching for Mastery

Why does it work?

How does it work?

Research evidence for Mastery

Is it for all schools?

Using materials to support teaching for mastery

What does it mean to master something?

- I know how to do it
- It becomes automatic and I don't need to think about it- for example driving a car
- I'm really good at doing it – painting a room, or a picture
- I can show someone else how to do it.

Mastery of Mathematics is more.....

- Achievable for all
- **Deep** and sustainable learning
- The ability to build on something that has already been sufficiently mastered
- The ability to reason about a concept and make connections
- Conceptual and procedural fluency

Teaching for Mastery

- The belief that all pupils can achieve
- Keeping the class working together so that all can access and master mathematics
- Development **of deep** mathematical understanding
- Development of both factual/procedural and conceptual fluency
- Longer time on key topics, providing time to go deeper and embed learning

A teaching for mastery lesson

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Teaching for Mastery

The lesson

Relevant Tasks
and Activities

Fluency Practice

Key Questions

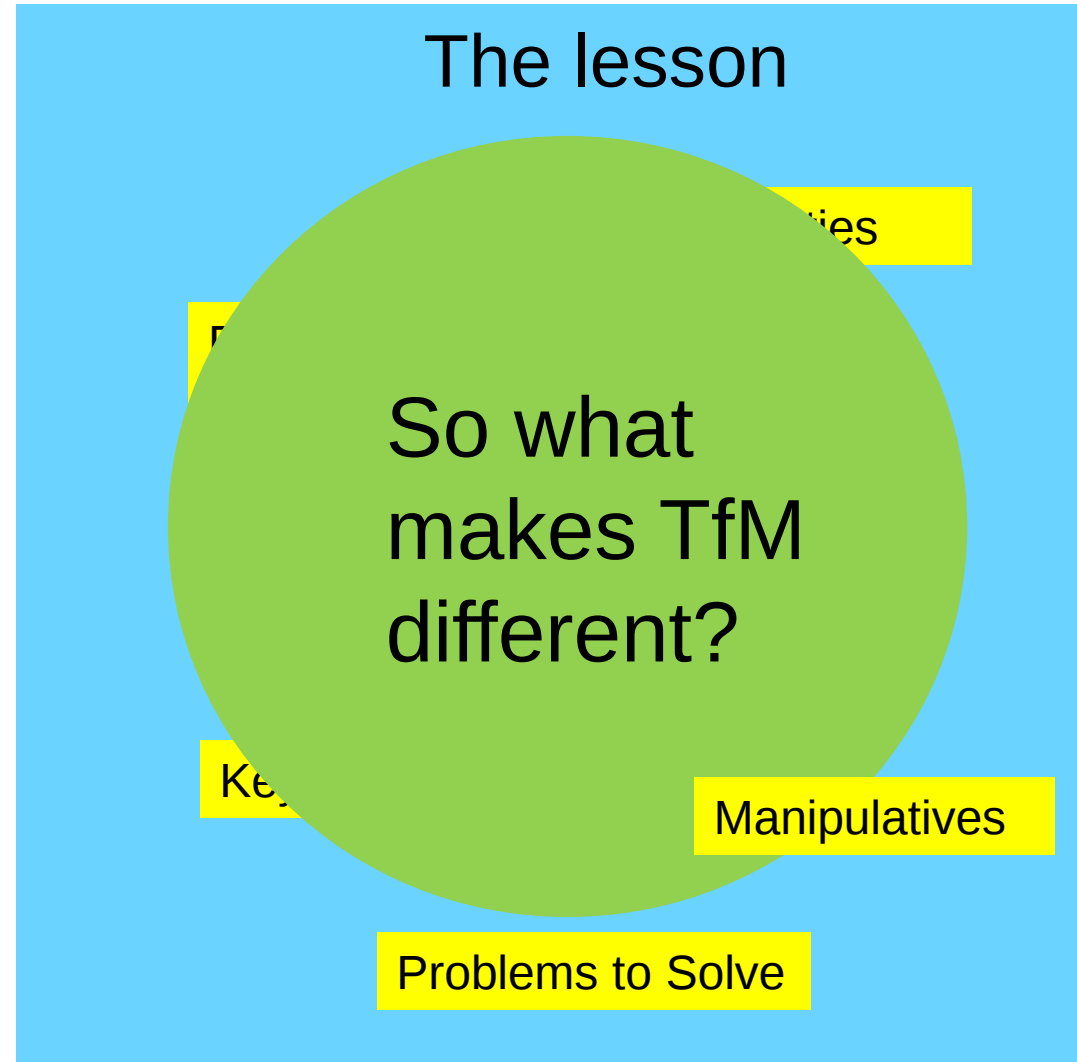
Manipulatives

Reasoning Activities

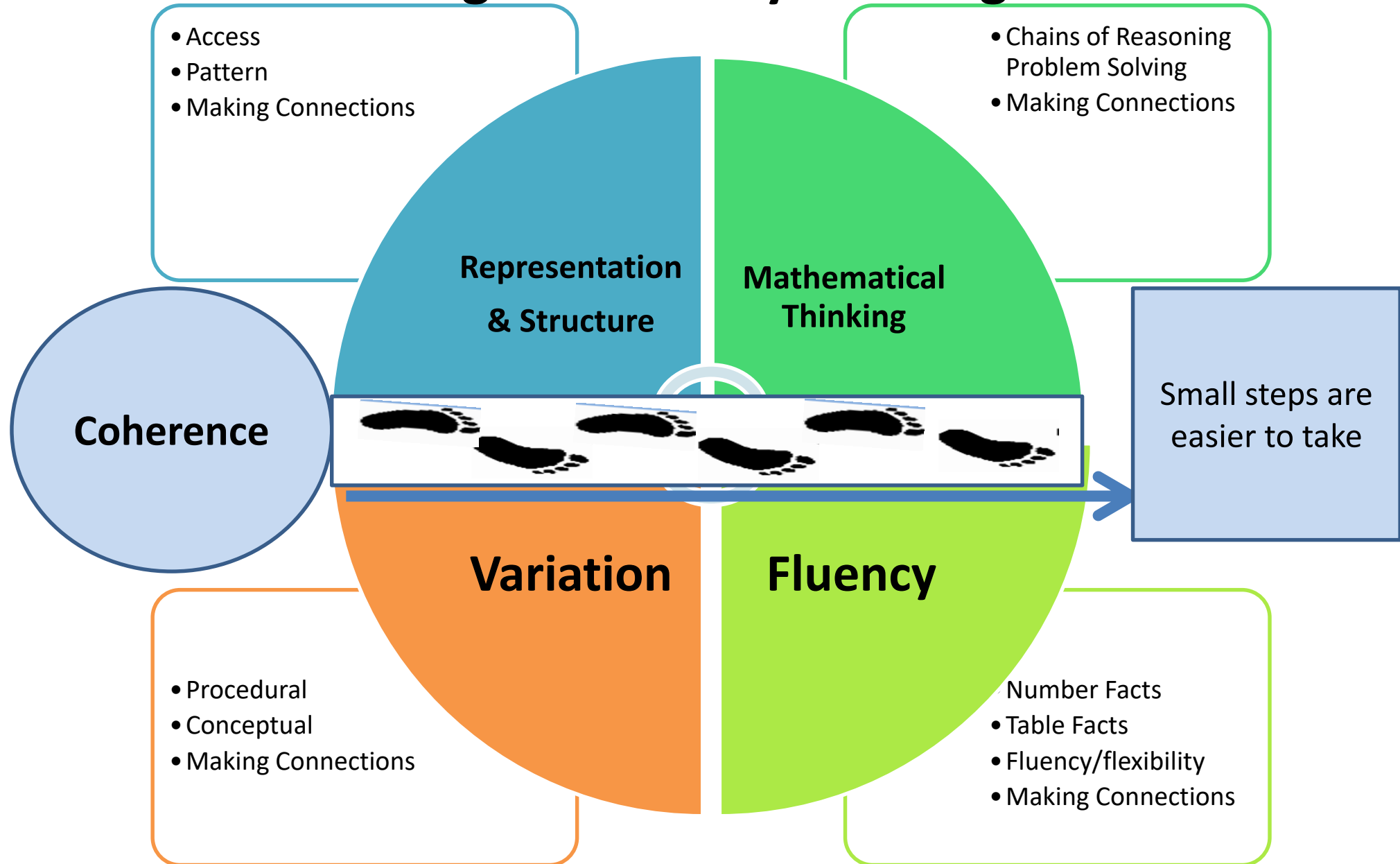
Challenge

Problems to Solve

Teaching for Mastery



Teaching for Mastery: Five Big Ideas



So what does a teaching for mastery lesson look like?

I am sure we all have ideas and many of us are doing it already. But there is still more to learn – teaching for mastery is about continual improvement.

We will engage in a lesson together

A Mastery Lesson

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Mr Williams has 4 packs of coloured whiteboard markers.

There are 8 pens in each pack.

How many pens are there altogether?



Partner B



Show the structure with counters

Partner A

A photograph of a whiteboard with handwritten equations. The equations are: $4 \times 8 = 32$, $8 + 8 + 8 + 8 = 32$, and $8 \times 4 = 32$.

Write an equation to represent it

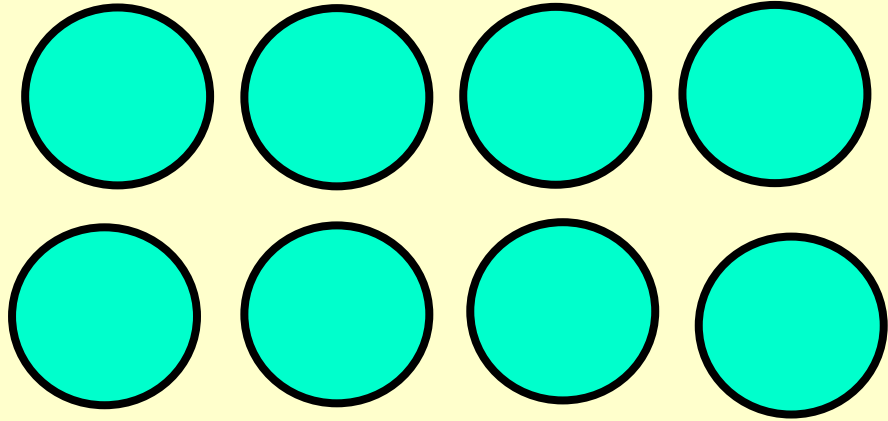
★ Write more than one equation

$$4 \times 8 = 32 \quad 8 + 8 + 8 + 8 = 32$$

$$8 \times 4 = 32$$



One to one correspondence



One counter represents **one** pen

I can see 8 ones

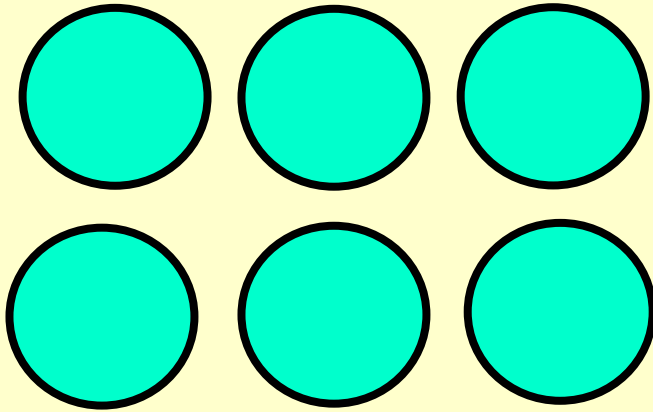
One to many correspondence



One counter represents **one** pack of (group) of pens

I can see 1 eight

One to one correspondence



One counter represents **one** pen

I can see 6 ones

One to many correspondence



One counter represents **one** pack of (group) of pens

I can see 1 six

Mr Williams gets out 3 packs of pens.
One pack has 6 pens, one has 8 pens and one has 12 pens.
How many pens altogether?



Partner A



Show the structure with counters

Partner B

$$6 + 8 + 12 = 26$$

Write an equation to represent it

Mr Williams gets out 3 packs of coloured markers.
He also gets out 5 packs of black markers.



The coloured packs have 8 pens in each. The black packs have 6 pens in each.
How many pens are there altogether?

Partner B

Show the structure with counters

Partner A

Write an equation to represent it

★ Write more than one equation.

3 packs of 8 and 5 packs of 6



How many can you see?



I can see 3 eights



I can see 5 sixes

How many can you see?



I can see 3 eights

$$3 \times 8$$



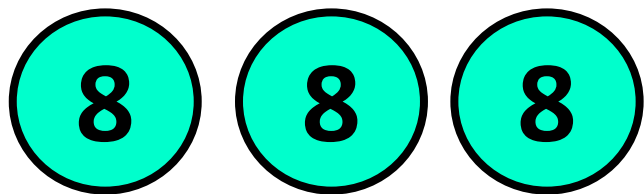
I can see 5 sixes

$$5 \times 6$$

I can see 3×8 and

$$5 \times 6$$

$$3 \times 8 + 5 \times 6$$



$$3 \times 8 + 5 \times 6 =$$

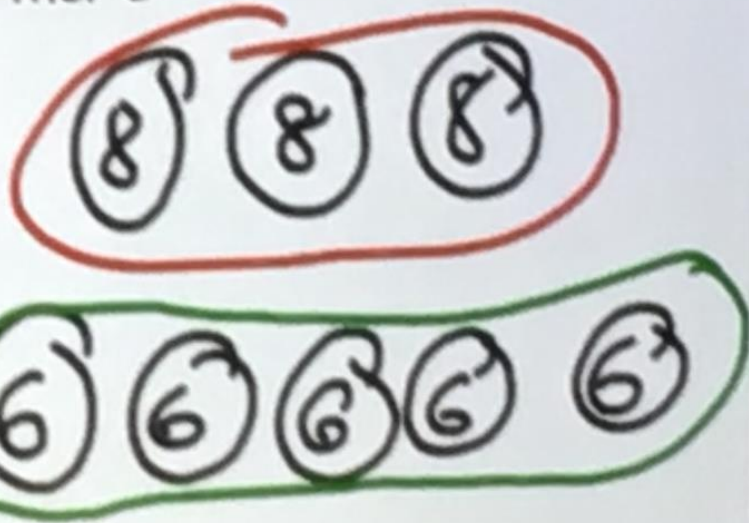
$$24 + 30 = 54$$

Mr Williams gets out 3 packs of coloured markers.
He also gets out 5 packs of black markers.



The coloured packs have 8 pens in each. The black packs have 6 pens in each.
How many pens are there altogether?

Partner B



Show the structure with counters

Partner A

$$8 + 8 + 8 + 6 + 6 + 6 + 6 + 6 = 54$$

$$3 \times 8 + 5 \times 6 = 24 + 30 = 54$$

Write an equation to represent it

Match the expressions to the images

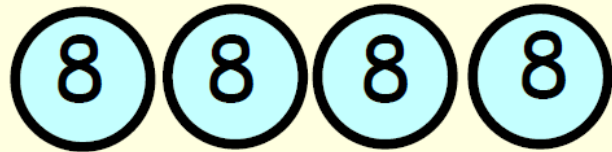
Different structures:

No repeating groups



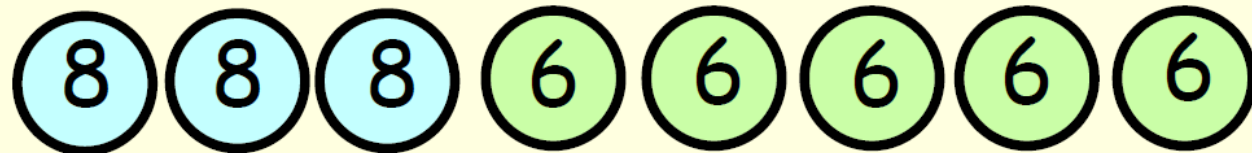
$$6 + 8 + 12$$

One repeating group



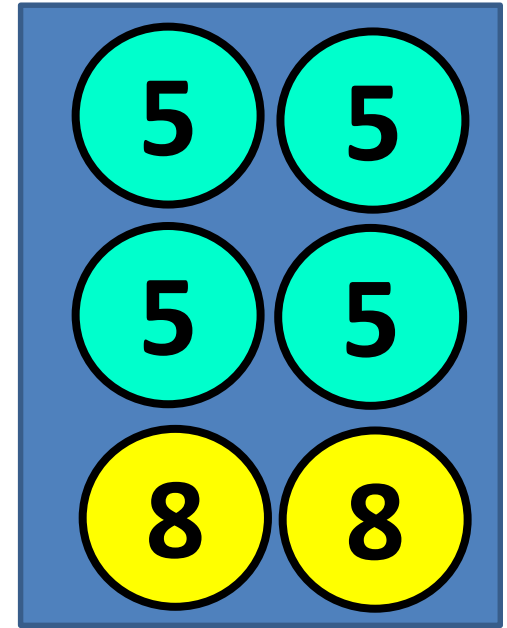
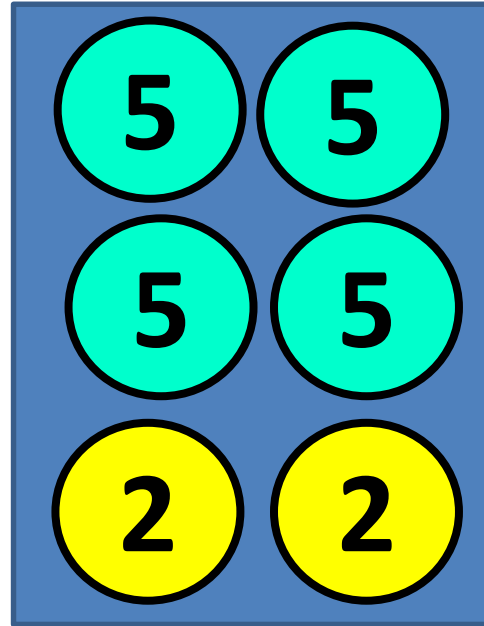
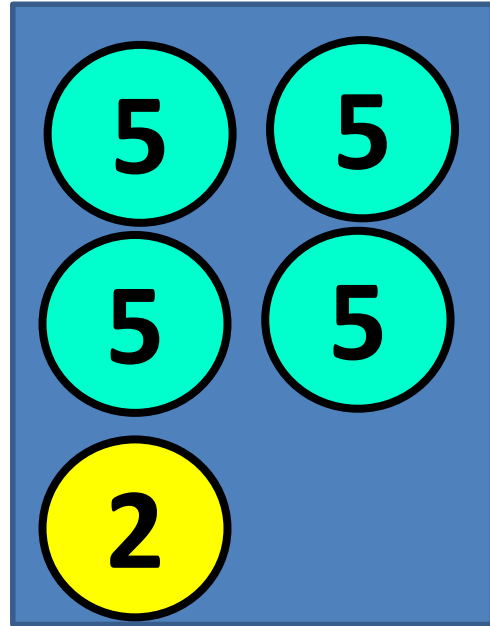
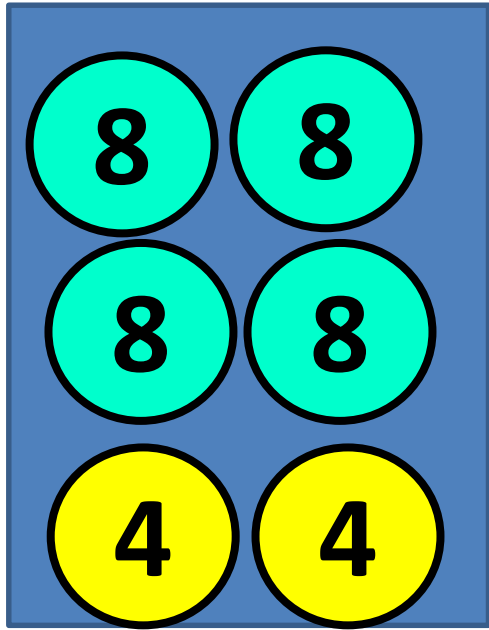
$$4 \times 8$$

Two (or more) repeating groups



$$3 \times 8 + 5 \times 6$$

Match each image to an expression



$$4 \times 5 + 2 \times 8$$

$$4 \times 5 + 2 \times 2$$

$$4 \times 5 + 2$$

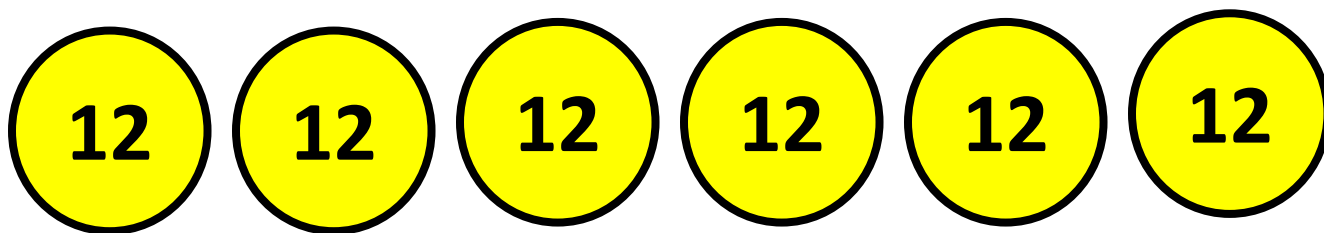
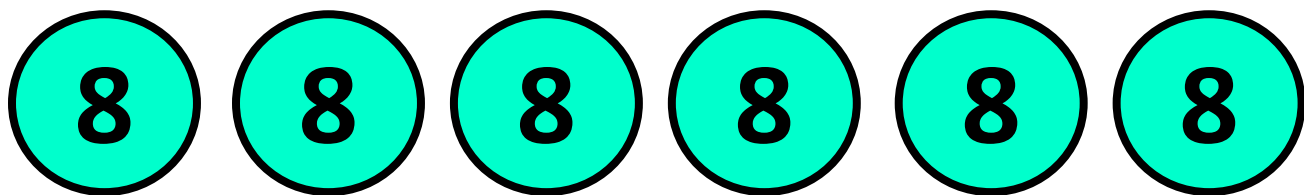
$$4 \times 8 + 2 \times 4$$

I buy 6 cakes ,
4 of them cost 60p each
2 of them cost 70p each
How much do they cost in total?

A shop sells bunches of flowers
for £8
And boxes of chocolates for £12
I buy 6 of each
Find a 'clever' way to calculate

I jog 8 km each day on Monday
to Friday
At the weekends I jog 15 km per
day
How far do I jog in total?

I buy 4 yellow highlighters at 8p
each and 2 pink highlighters at
8p each.
Find a 'clever' way to calculate



6 x 20

I jog 8 km each day on Monday to Friday

At the weekends I jog 15 km per day

How far do I jog in total?

The image shows a handwritten calculation on blue grid paper. At the top, the number 4 is written to the left of five circled 8s, and the number 2 is written to the left of two circled 15s. Brackets are drawn under each group of circled numbers. Below the first bracket is the number 40, and below the second is 30. These are added together to get 70. Below this, the text 'RWA:' is written, followed by the equation $5 \times 8 + 2 \times 15 =$.

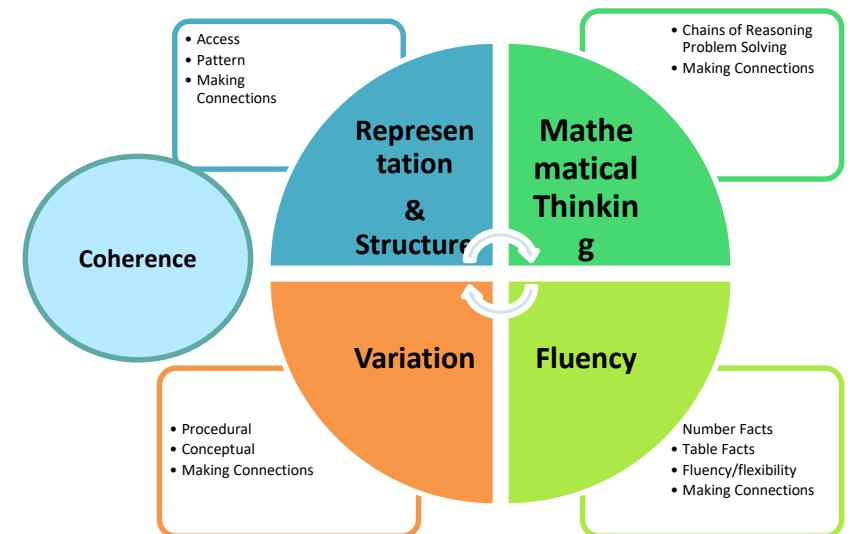
$$40 + 30 = 70$$

RWA:

$$5 \times 8 + 2 \times 15 =$$

Discussion: What did you notice?

- Sequencing of tasks – coherence
- Repetition and development of fluency
- Use of representations to draw out mathematical structure
- Opportunities to reason and make connections
- Inclusion of all learners

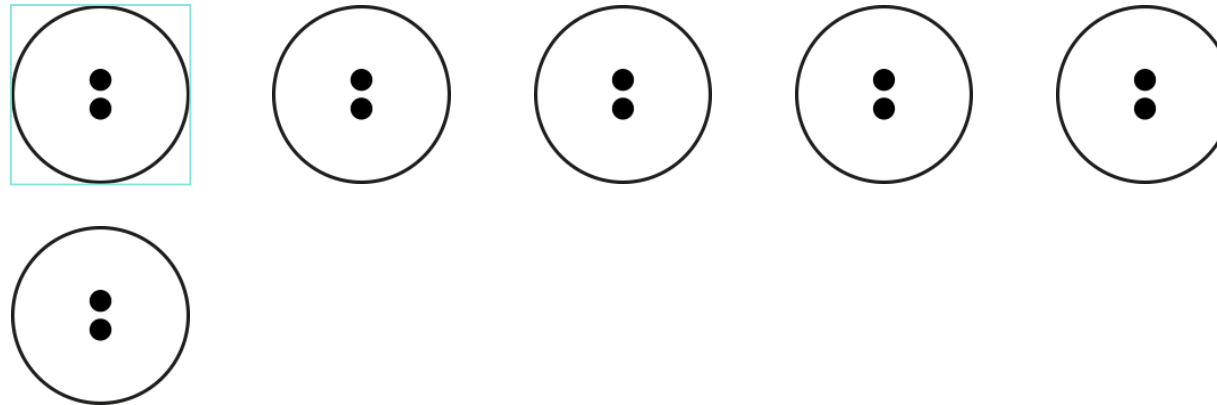


Exposing Mathematical Structure and Unitising

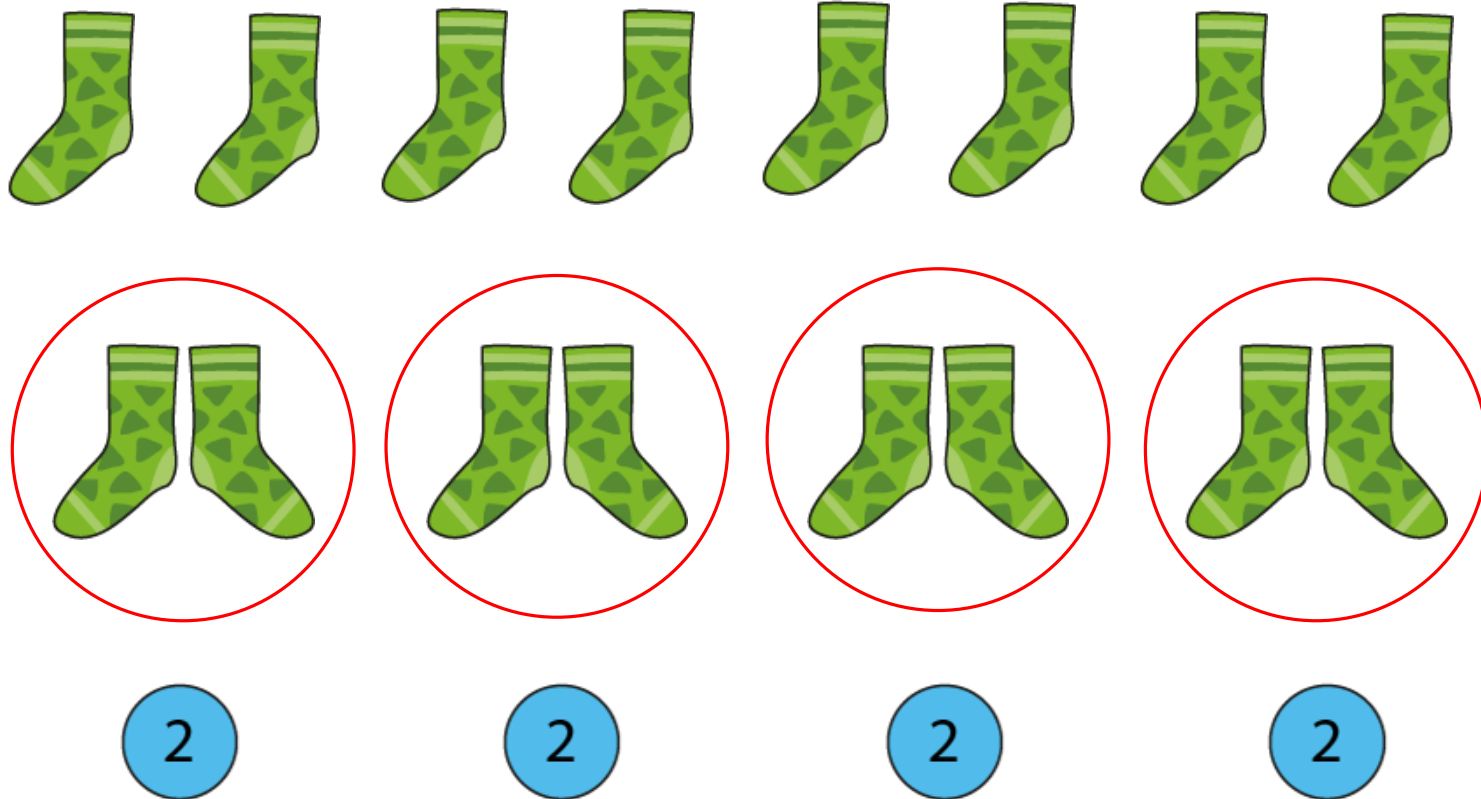
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How many dots are there?
Count in groups of two.

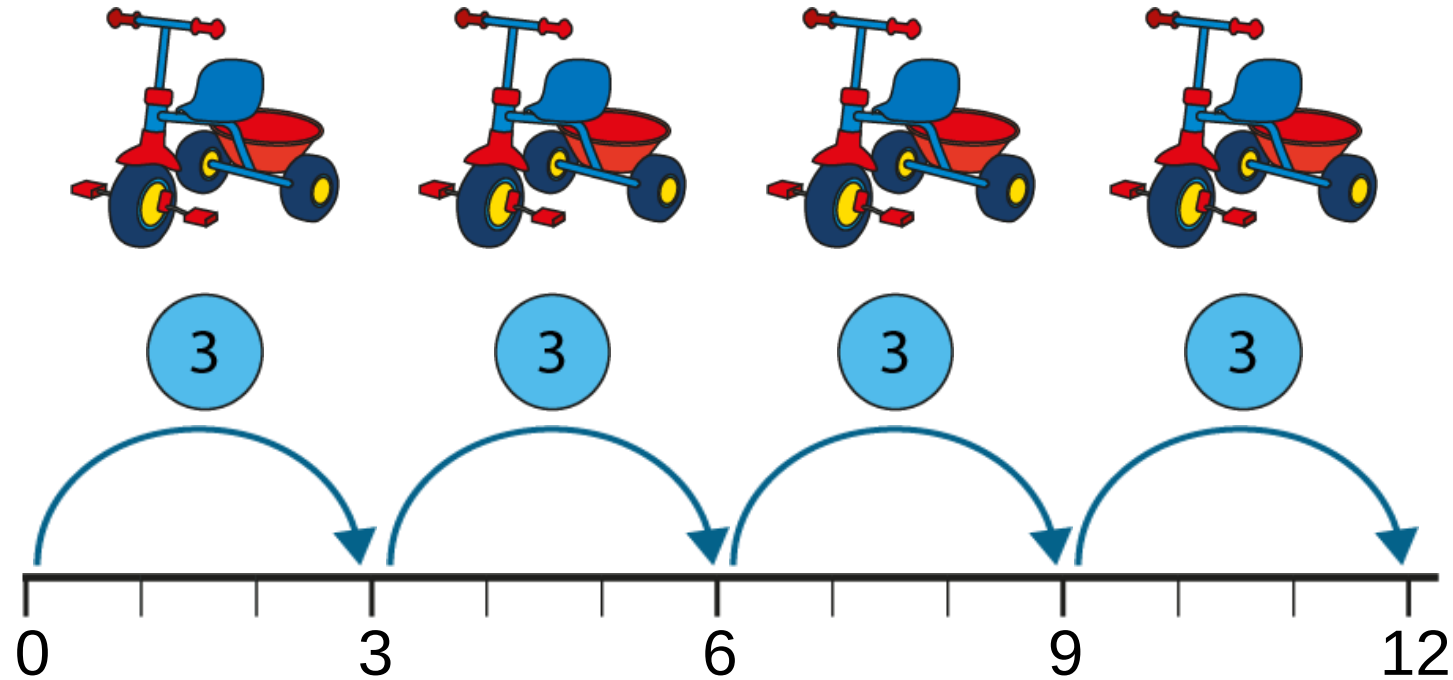


8 socks, how many pairs?



8 is divided into groups of 2. There are 4 groups.
There are 4 groups of 2 in 8.

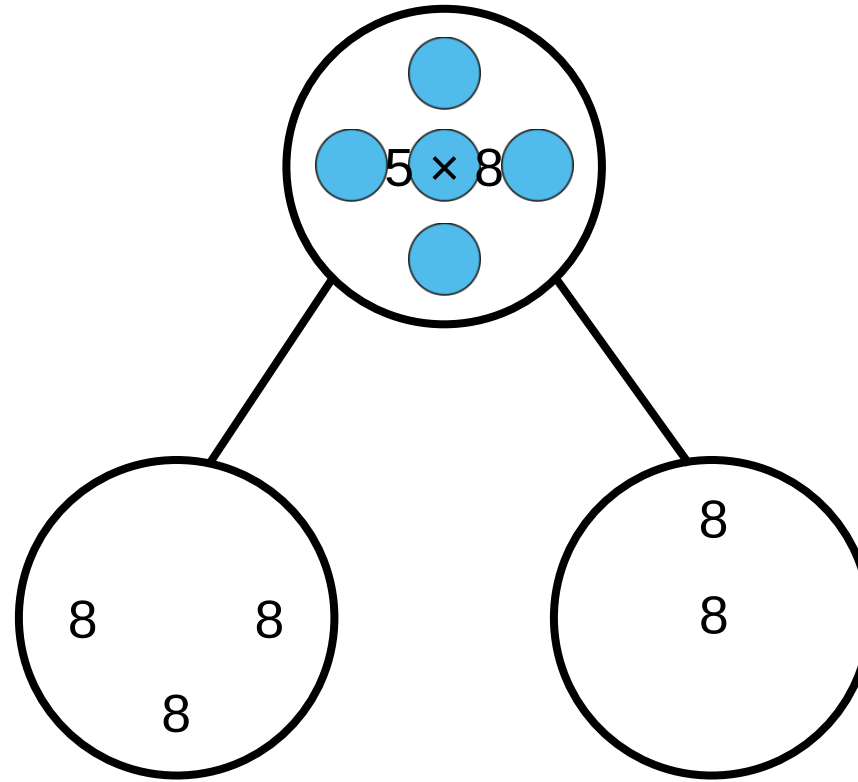
How many wheels? Count in groups of 3.



There are 12 wheels.

$$4 \times 3 = 12$$

$$3 \times 4 = 12$$



$$5 = 3 + 2$$

$$5 \times 8 = 3 \times 8 + 2 \times 8$$

$$= 24 + 16$$

$$= 40$$

How does it work?

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Pupils take the same path at the same time.



We now think of
inclusion in a
different way

The smaller the distance from the existing
knowledge and the new learning,
the greater the success

Gu, L. (1994).



Meeting the Needs of Higher Attaining Pupils



- Breaking down the curriculum into smaller steps results in greater rigour and depth of understanding.
- Moving more slowly means there is greater time to think, make connections and apply mathematics.
- There are higher expectations in terms of language and explanations.
- The concepts children need to learn are the same for all pupils.
- The teaching of the concepts should be the same for all but the outcomes in terms of application may be different

Two Stories of High Attaining Pupils

Bethany Year 1

Lesson on difference

Liam Year 3

Lesson on Fractions

Research evidence

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Pudian

*By putting blocks or stones together as a Pudian, a person can pick fruit from a tree which cannot be reached without the Pudian” .
(Gu 2004 p340)*



Small focused steps

It was noticed that when the potential distance is too long, a majority of students have difficulties in approaching the new knowledge, which we conjectured was due to heavy cognitive load (Gu, 1994).



A teaching for mastery lesson should start at the point where all children can access and then move the class forward together

Small Steps – drawing on cognitive science

It's important to spread out learning over many days, his work shows. That means learning a little bit at a time. Doing so allows links between neurons to steadily strengthen. It also allows glial cells time to better insulate axons.

Even an “aha!” moment — when something suddenly becomes clear — doesn't come out of nowhere. Instead, it is the result of a steady accumulation of information

<https://www.sciencenewsforstudents.org/article/learning-rewires-brain>

Avoiding overload

Working memory is where we engage in thinking, make sense of and connect what we are learning. Cowan (2010) estimates that the number of items that can be held and processed at any one time is approximately four. If there are too many things to think about at once, then the brain is on overload and cannot think about anything with clarity.

Teaching for mastery breaks down the mathematics into “thinkable chunks” and joins them together

The role of memorisation

Memory is more than a repository for factual knowledge. It organises and connects information, representing both knowledge and meaning.

Knowing how learners develop coherent structures of information has been particularly useful in understanding the nature of organised knowledge that underlies effective comprehension and thinking. (Bransford 1999)

The careful ordering, focus and repetition provided through the lesson supports embedding in the long term memory

Staying focused

The fact that the material you are dealing with has meaning does not guarantee that the meaning will be remembered. If you think about that meaning, the meaning will reside in memory. If you don't, it won't.
(Willingham 2003)

We remember what we think about. A teaching for mastery lesson is designed to provoke thinking and make meaning. Making meaning will help us to remember

Remember these letters

PFB IJFKIBMDNA

With thanks to
David Thomas

Easier now?

P FBI JFK IBM DNA

Retrieval builds storage strength

- When a computer reads a CD, it doesn't alter the information on the CD
- Our minds are different – when we retrieve a memory our brains rewrite it before putting it back in our long-term memory
- It rewrites it by:
 - Increasing its storage strength
 - Adding connections
 - Correcting misconceptions



"I've thought about this again and used it to help with other things, it must be important!"

With thanks to
David Thomas

Retrieval through review and testing (quizzing)

What did we learn yesterday?

Testing is not a bad thing!

As long as it is low stakes

Getting 'stuff' out of the long term memory and putting it back strengthens retrieval

If we get it out and it is incorrect we can put it back corrected

It's not an assessment strategy, but a learning strategy.

All Children are Mathematicians

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Intelligent Practice

$$5 \times \begin{array}{|c|} \hline 1 \\ \hline 3 \\ \hline 5 \\ \hline 7 \\ \hline 9 \\ \hline \end{array} = \begin{array}{|c|} \hline \\ \hline \\ \hline \\ \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline 0 \\ \hline 2 \\ \hline 4 \\ \hline 6 \\ \hline 8 \\ \hline \end{array} \times 5 = \begin{array}{|c|} \hline \\ \hline \\ \hline \\ \hline \\ \hline \end{array}$$

5

6

9

She makes a 2-digit number and a 1-digit number.

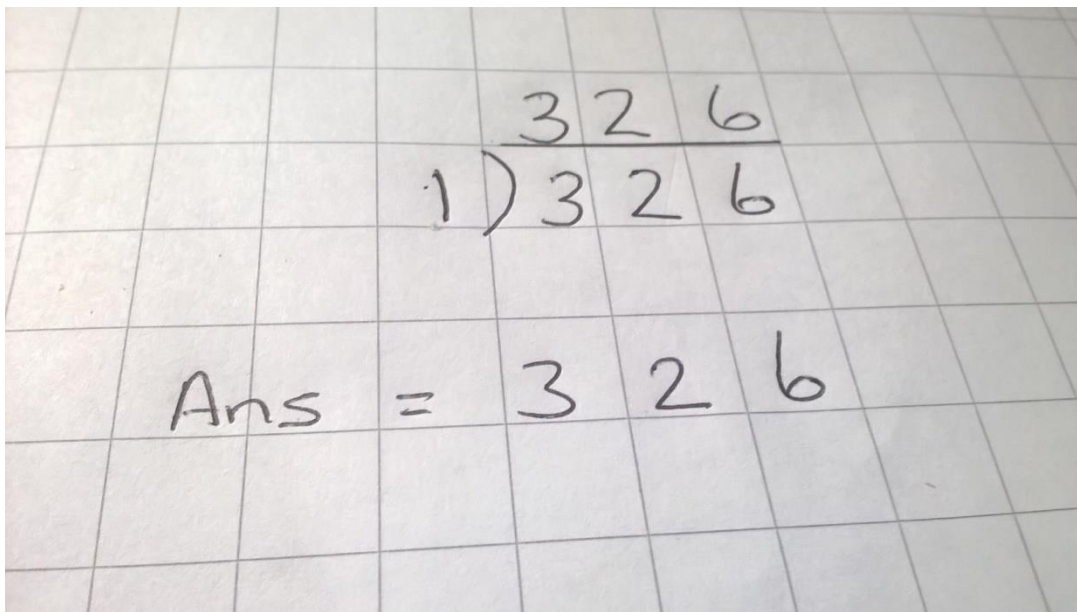
She multiplies them together.

Her answer is a **multiple of 10**

What could Chen's multiplication be?

Is this Mastery?

$$326 \div 1 =$$



A photograph of a student's handwritten work on a grid background. The student has written the long division of 326 by 1. The divisor '1' is written to the left of the dividend '326', which is underlined. The quotient '326' is written above the dividend. Below the division, the student has written 'Ans = 326'.

$$\begin{array}{r} 326 \\ 1 \overline{) 326} \end{array}$$

Ans = 326

What Mastery isn't

Thinking about relationships

21

$$5,542 \div 17 = 326$$

Explain how you can use this fact to find the answer to 18×326

$$17 \times 326 = 5,542$$

$$18 \times 326 = 5,542 + 326$$

How might children respond to this question?
What is the best response?

A deep body of knowledge

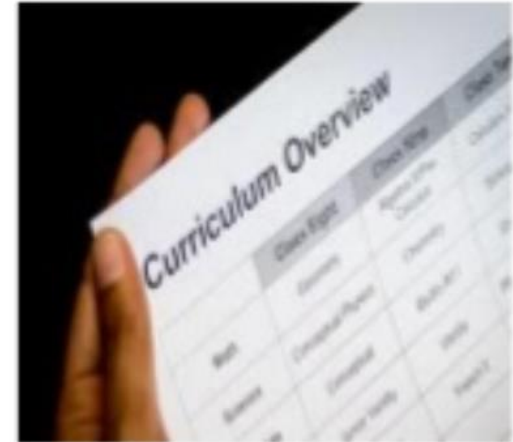


The importance of the **curriculum**



'If [children's] entire school experience has been designed to push them through mark-scheme hoops, rather than developing **a deep body of knowledge**, they will struggle in later study.'

Amanda Spielman, at the launch of Ofsted's Annual Report
2016/17





Has the content of the curriculum been
learned long term?



'Learning is defined as an alteration in
long-term memory. If nothing has altered
in long-term memory, nothing has been
learned.'

Sweller, J., Ayres, P., & Kalyuga, S. (2011). Cognitive load theory (Vol. 1). Springer Science & Business Media.



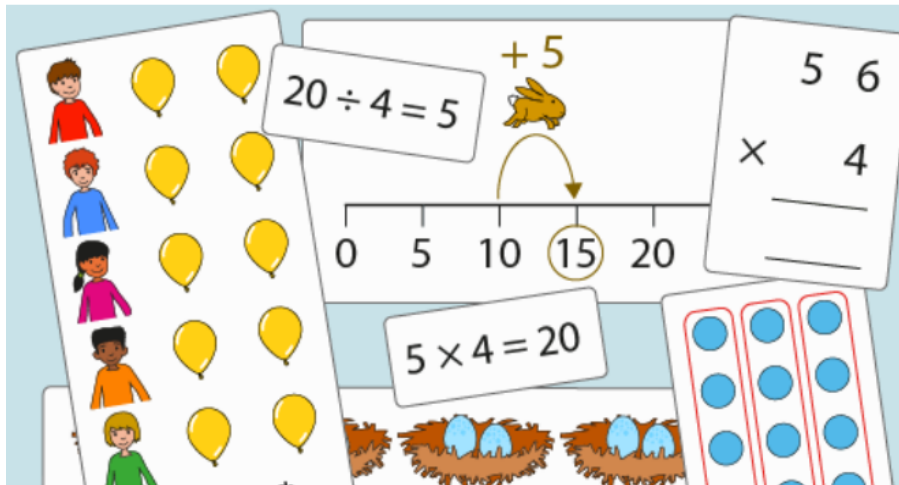
Concepts that matter when discussing the curriculum



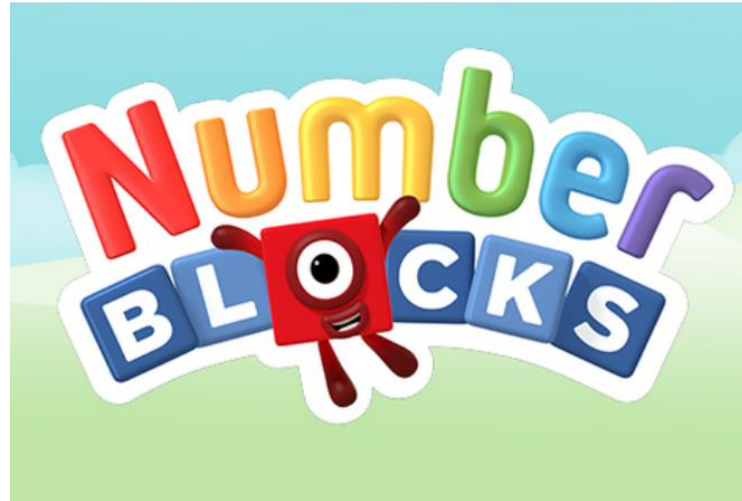
- Progress means knowing more and remembering more.
- Knowledge is generative (or 'sticky'), i.e. the more you know easily you can learn.
- Knowledge is connected in webs or 'schemata'.
- Vocabulary size relates to academic success, and schooling is crucial for increasing the breadth of children's vocabulary.



Resources



[Mastery PD Materials](#)

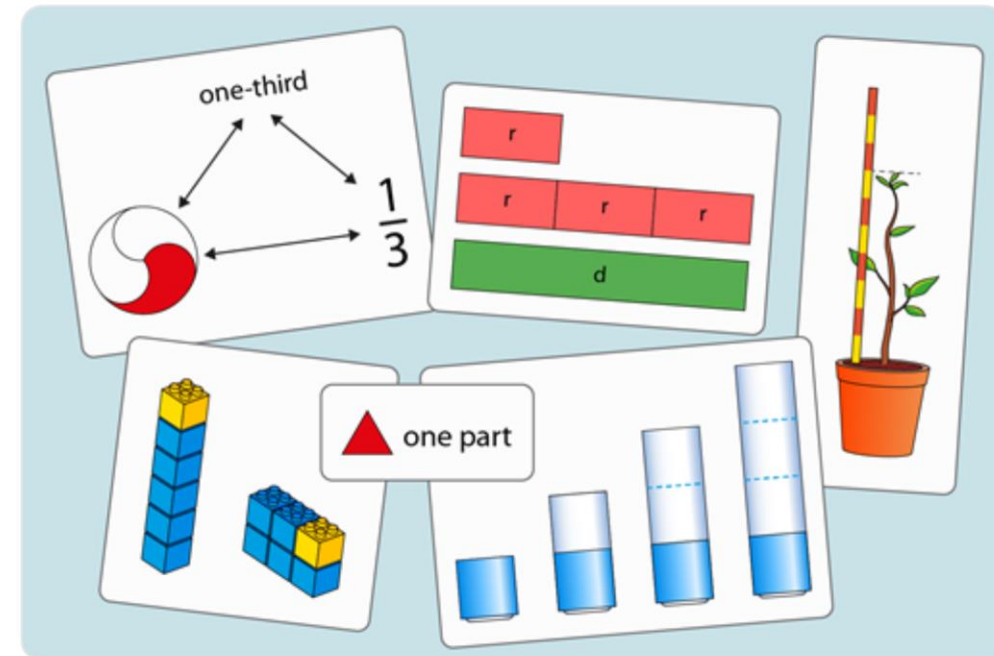


[Numberblock Support Materials](#)

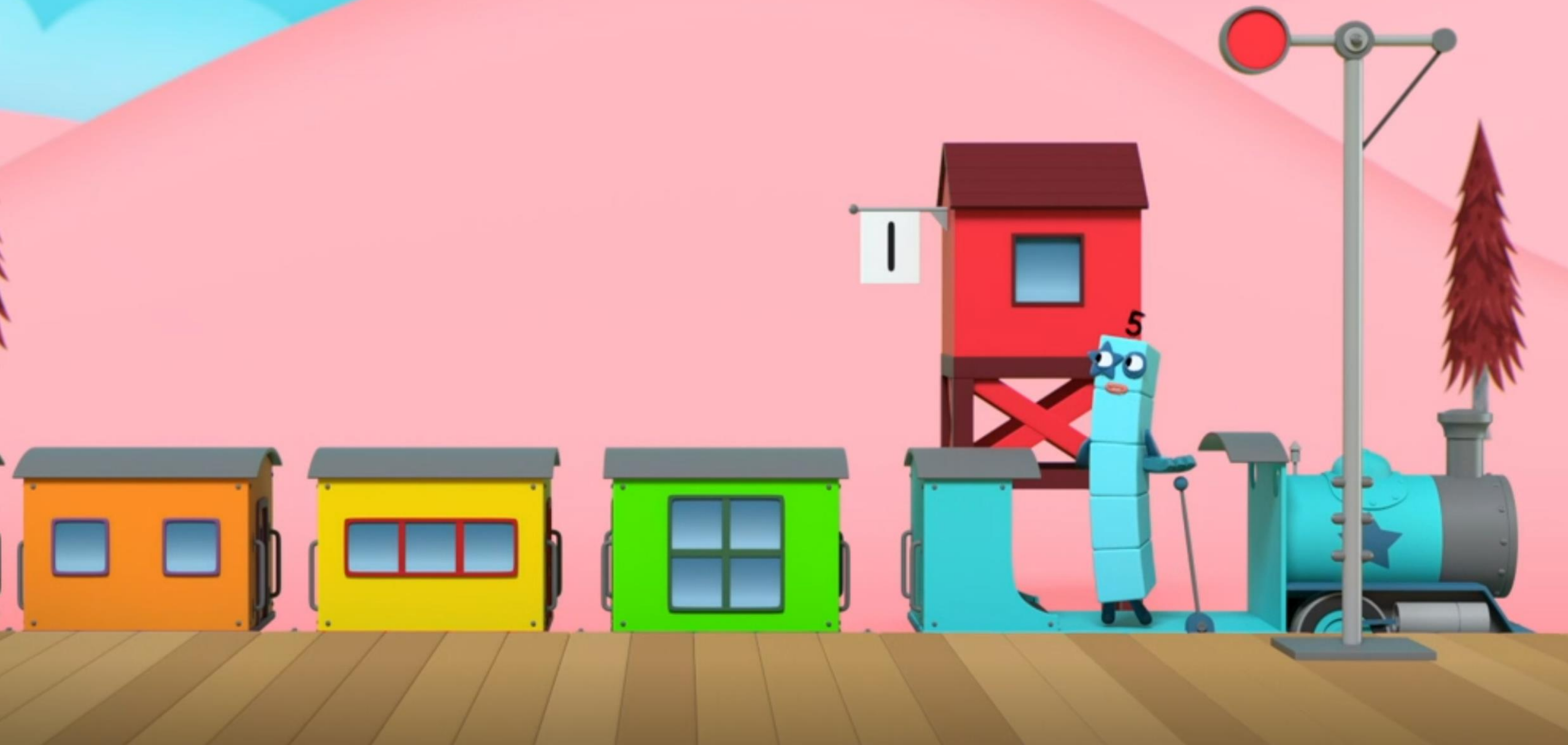
Resources: The Mastery PD Materials

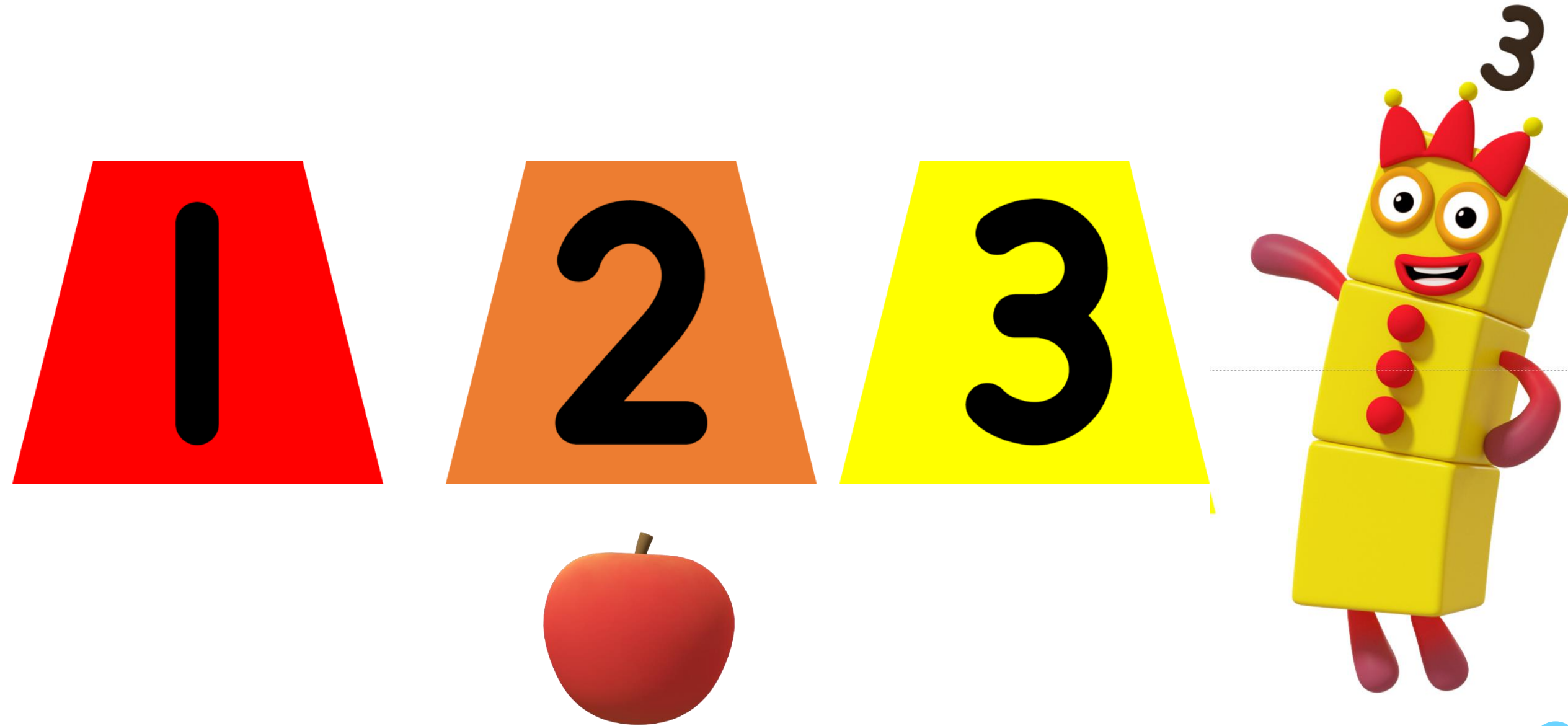
Purpose:

- To support primary teacher subject knowledge – demonstrating a small step coherent journey that transfers into classroom practice
- Exposes relationships and structures
- Provides a rich curriculum that develops children as mathematicians



Composition of 5











Teaching for Mastery

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1. We ALL
start the
journey
TOGETHER

2. Some children
will need a little
additional
support along
the way

3. Some children, who feel confident, will
be let loose. They'll be able to explore
deeper into the woods, before returning
to the group to continue on with the
journey.

5. Children will
not be left
behind alone
and isolated.

4. Children will not
be racing off ahead
on a different
journey.



Martin Adsett
Mastery Specialist

We're Going on a Maths Hunt

Teaching for Mastery

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We're **ALL** *Going on a* **Maths Hunt**

We're **Not Scared!**